NEW ASTEROIDS.—M. Charlois discovered the asteroid (298) on September 9. On October 3, what was supposed to be the same body was again observed by Charlois. Dr. Palisa also observed an asteroid near the computed position of (298) on October 11. It has since been found that the observations of October 3 and 11 must refer to another asteroid, and, as Dr. Palisa discovered (299) on October 11, the one thought to be identical with (298) will be reckoned as (300). M. Charlois rediscovered (298) on November 14. Dr. Palisa added (301) the list on November 16 (*Astronomische Nachrichten*, 3006).

The large number of asteroids now known renders it impossible for the Berlin Computing Bureau to furnish all the data for their observation. The editors of the *Berliner Jahrbuch* have therefore decided only to furnish predictions for the following asteroids:—

(1) Those that approach near the earth, and are therefore well adapted to the determinations of parallax.

(2) Those whose near approach to Jupiter renders them useful for determining his mass.

(3) Those remarkable for a period having a closely commensurable ratio to that of Jupiter; such orbits being of the highest importance in the theory of absolute perturbations.

(4) Those that are of value for photometric work on account of their considerable brightness.

ZONA'S COMET (*e* 1890).—Dr. Bidschof gives the following elements and ephemeris of this comet in *Astronomische Nachrichten*, 3006:—

T = 1890 July 25·051 Berlin mean time.

$$\left.\begin{aligned}\omega &= 321^\circ\ 58'\\ \text{☊} &= \ \ 84\ \ 45\\ \iota &= 153\ \ 28\end{aligned}\right\}\text{ Mean Eq. 1890.}$$

Perihelion distance = 1·8996 (earth's mean distance = 1).

Ephemeris for Berlin Midnight.

1890.	R.A.	Decl.	Distance in terms of the earth's mean distance.
	h. m.	° ′	
December 4	3 47·0	+ 34 50	1·53
,, 8	3 26·8	+ 34 19	1·59
,, 12	3 8·7	+ 33 36	1·65

The comet is, therefore, in the constellation Perseus, which is in the south about 10 p.m. It passed perihelion about the end of July, and is increasing its distance from the earth. M. Bigourdan, of Paris Observatory, describes the comet as "très faible (grandeur 12·5–13), et se présentait sous l'aspect d'une petite tache blanche, ronde, de 1′ environ de diamètre, avec condensation central assez diffuse et d'aspect un peu grande."

THE SCIENTIFIC WORK OF JOULE.[1]

PROF. DEWAR commenced by remarking that the Royal Institution had been so closely identified with the great workers in physical science that it was impossible to allow the work of Joule, whose researches had produced as marked a revolution in physical science as Darwin's in biology, to pass without recognition in the present series of Friday evening discourses. Sir William Thomson, as Joule's friend and fellow-worker to the last, had been invited to undertake the duty, and had agreed to do so; but at the last moment had been compelled to decline by reason of important official duties in Scotland, and the task had consequently devolved upon him.

Having given a brief account of Joule's parentage, early life, and education, Prof. Dewar reviewed, as fully as time would permit, his scientific work, which extended over about forty years, and was represented by 115 original memoirs. The first period (1838-43) was distinguished as that in which Joule educated himself in experimental methods, chiefly in connection with electricity and electro-magnetic engines. This work led him in 1840 to his first great discovery, the true law governing the relation between electric energy and thermal evolution, which enabled him later on to account for the whole distribution of the current, not only in the battery in which it is produced, but in conductors exterior to it. Joule was thus led to take up the study of electrolysis. Faraday had already made the discovery that electrolytic bodies could be split up into equivalent proportions by the passage of the same electric current; Joule saw that there would be great difficulty in finding out the distribution of the current energy, and accounting for the whole of it. After a laborious research he succeeded in showing that during electrolytic action there was an absorption of heat equivalent to the heat evolved during the original combination of the constituents of the compound body. The prosecution of his electrical researches rapidly brought Joule on the road to his great discovery of the mechanical equivalent of heat, it being clear from a footnote to a paper dated February 18, 1843, that he already had well in hand the study of the strict relations between chemical, electrical, and mechanical effects.

In working out these laws, it was to be remarked that Joule—in common with most inventors and seekers after new scientific truths—chose perhaps the most difficult means that could have been selected; and in looking back at his work in the light of present knowledge, it seemed simply astounding that he should have succeeded so completely as he did. The original coil used by Joule for the mechanical determination of heat (kindly lent for the occasion by Prof. Rücker) was shown, and the course of the experiment explained. The vast difficulties which Joule had to overcome in order to prove that there was a definite, permanent, and persistent relation between the amount of mechanical energy expended and the heat produced were commented on; the thermal effects being produced not directly but through the medium of an electric current varying in intensity, and calculations having to be made not only for these fluctuations, but for the effects of radiation, the movement of the air, and other indirect complications. The very small increment of heat to be measured obliged Joule to use thermometers of great delicacy, and these he had to devise and construct himself. One of the thermometers so used was exhibited.

Working in this way, Joule was able, by the end of July 1843, to state definitely that the amount of heat capable of increasing the temperature of a pound of water by 1° F. was equal to, and might be converted into, a mechanical force capable of raising 838 lbs. to the height of 1 foot. Soon afterwards he attained almost identical results by a more direct method—the friction of water passing through small tubes—which gave him 770 foot-pounds per unit of heat.

It was impossible, said the lecturer, to thoroughly appreciate Joule's work without glancing at the early history of the subject; and when one did so it was amazing to find how near men of the stamp of Rumford, Davy, and Young had been to Joule's great discovery, and yet missed it. Count Rumford was the first to clearly define the relation between the constant production of heat and loss of movement by frictional motion. He proved that the amount of heat produced by friction was continuous, and apparently unlimited; but he did not think of measuring the relation between the mechanical energy expended and the amount of heat produced. Alluding to the results obtained from this apparatus, the lecturer said that Count Rumford might have shown that in his experiments the heat produced was proportional to the time of working, and so obtained a result capable of being expressed in horse-power. The value so deduced from Rumford's experiments is not far removed from Joule's first number.

The experiments commenced by Count Rumford were carried on by Davy, at that time working with Beddoes at Bristol; and led to one of the most remarkable essays on heat of that period, which disposed for ever of the theory of the separate existence of caloric. Taking two pieces of ice on a cold day, Davy mounted them so that they could be rotated against each other with frictional pressure, the effect being that the pieces of ice were melted, the water so produced having a much higher specific heat than the original ice. To guard against the possibility of heat being conveyed to the frictional apparatus by the surrounding air, Davy made an experiment *in vacuo*, isolating the apparatus by means of ice; and found that under such conditions sufficient heat could be produced to melt wax placed in the receiver. The lecturer here showed an experiment illustrating the production of water by the friction of two pieces of ice *in vacuo*, under conditions of temperature much much severe than those of Davy's experiment.

Following Davy, Young devoted a great deal of attention to the subject, and by 1812 he and Davy had quite changed their opinions, and had adopted the view that heat and motion were convertible effects.

Having by July 1843 assured himself of the principle of his discovery, Joule now devoted his attention to the elaboration of methods of working, modifying, and repeating experiments in various ways, but always approaching nearer and nearer to exactness, as shown by the following table of results:—

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[1] Abstract of the Friday evening discourse delivered at the Royal Institution of Great Britain on January 24, 1890, by Prof. Dewar, F.R.S.

Joule's Values of the Mechanical Equivalent of Heat.

		Kilogramme metres.
Magneto-electric currents	1843 ...	460
Friction of water in tubes	,, ...	424·6
Diminution of heat produced in a battery current when the current produces work	,, ...	499
Compression of air	1845 ...	443·8
Expansion of air	,, ...	437·8
Friction of water	,, ...	488·3
,, ,, ,, ...	1847 ...	428·9
,, ,, ,, ...	1850 ...	423·9
,, ,, mercury	,, ...	424·7
,, ,, iron	,, ...	425·2
Heat developed in Daniell's cell ...	,, ...	419·5
,, ,, in wire of known absolute resistance	1867 ...	429·5
Friction of water in calorimeter ...	1878 ...	423·9

Prof. Dewar here exhibited and explained the action of the original calorimeter used by Joule. It was seen to consist of a set of vanes which were made to revolve in water by the falling of known weights through a definite and known height, the heat produced being due (after making the necessary deduction for the friction due to the momentum of the weights) entirely to the friction of the fluid. It was found that, whatever fluid was employed, the same definite results were obtained—a production of heat in the liquid bearing a constant relation to the unit of mechanical energy expended. The extreme delicacy of Joule's apparatus, and the marvellous accuracy of his observations, were shown by the fact that working with weights of 29 pounds each, and repeating each observation 20 times, the total increase of temperature did not exceed half a degree Fahrenheit. In contrast to this the lecturer showed, by means of apparatus kindly lent by Prof. Ayrton, the method now employed for repeating Joule's work and arriving at substantially the same results by much simpler means.

While continuing to work intermittently at his great discovery, Joule employed himself in the following years in elaborate investigations bearing upon the point of maximum density of water, specific gravity, and atomic volumes. An illustration of his method of determining maximum density was given by means of two large cylinders filled with water and connected by a narrow channel in which was placed a floating indicator. It was shown that the slightest variation in density of the water of either cylinder—variations far beyond the scope of the most delicate thermometer—set up currents which were immediately detected by the movement of the indicator, and that by this means it was quite possible to ascertain the exact temperature at which water attained its maximum density.

Joule's determinations of atomic volumes were marvellous at the time they were made, and were still interesting. Illustrations of his work in this direction were given by means of a solution of sugar, which was seen to occupy practically the same space as was occupied by an amount of water exactly equivalent to that combined in the carbohydrate, the carbon hypothetically combined with the water to form the sugar appearing to make no sensible difference to the volume; and in contrast to this was seen the enormous difference in volume brought about by dissolving two equal portions of soda carbonate, one portion being ordinary hydrated crystals, and the other portion being anhydrous, in equal volumes of water.

Joule's last great research was carried out conjointly with Sir William Thomson, and occupied nearly ten years of laborious inquiry. Its chief object was to prove that in compressing a gas the amount of heat produced is equivalent to the work done, and independent of the mere fact of the approach of the particles. But Joule was desirous of amplifying the inquiry, and in fact the work might be divided into three sections: (1) the study of gases passing through narrow apertures; (2) the velocity attained by bodies passing through the air; and (3) the temperature ultimately attained by such moving bodies. With respect to (2) and (3), it was shown that a body rotating in the air at the rate of about 150 to 180 feet per second increased in temperature by nearly 1° F., and that this increase of temperature was definite for a given velocity, and independent of the size of the moving mass and the density of the gaseous medium. With regard to (1), the relation of gaseous pressure and volume to temperature, the researches of Regnault had already shown that the simple law of Marriotte and Boyle could not stand by itself; and Joule sought to modify it by the study of gases passing through very small tubes or porous bodies. The investigations were carried out at Manchester on a large scale, and were assisted by a Government grant. Steam engines were employed to maintain a current of gas at a constant temperature and pressure through long coils of pipe placed in water tanks. They proved that any difference of temperature in the gas brought about in its passage through the porous body must be due to work done by it, and that this difference of temperature varied for different gases, according to their constitution. Working under the same conditions, hydrogen was shown to be reduced a small amount in temperature, air somewhat more (about 0°·3), and carbonic acid a much greater amount. A repetition of Joule and Thomson's experiment was shown by means of a 100-feet coil of lead pipe, compressed hydrogen, air, and carbonic acid gas being employed, and the original results verified in each case. The effect of this research was to enable Joule and Thomson to formulate a great improvement on the gaseous laws; for, instead of the product of the volume and pressure being strictly proportional to the absolute temperature, as it had been hitherto believed, they found that a new term was involved, which is equivalent to a constant divided by the absolute temperature and multiplied by the volume.

In conclusion, Prof. Dewar read the following letter, which he had received from Sir Lyon Playfair in response to his request for some reminiscences of Joule:—

January 20, 1890.

DEAR DEWAR,—You ask for some of my memories of Joule from 1842 to 1845, when I was Professor of Chemistry at the Royal Institution in Manchester. The great Dalton died in the autumn of 1844, and had long been President of the Manchester Philosophical Society. He naturally gave impulse to the study of science in that town, where there was an active band of young workers in research.

Joule was, even then, foremost among these; and the names of Binney, Williamson, Schunck, Angus Smith, Young, and others show that the spirit of scientific inquiry was active. We were also stimulated by the fact that Baron Liebig and Bunsen came to pay me visits during that time; they were men to excite research.

Joule was a man of singular simplicity and earnestness. We used to meet at each other's houses at supper, to help the progress of our work by discussion. Joule was an earnest worker, and was then engaged on his experiments on the mechanical equivalent of heat. He took me to his small laboratory to show me his experiments, and I, of course, quickly recognized that my young friend the brewer was a great philosopher. We jointly worked upon questions of far less importance than his great central discovery, but he was equally interested. I was very anxious that he should devote his life to science, and persuaded him to become candidate for the Professorship of Natural Philosophy at St. Andrews. He was on the point of securing this, but his slight personal deformity was an objection in the eyes of one of the electors; and St. Andrews lost the glory of having one of the greatest discoverers of our age.

When Joule first sent an account of his experiments to the Royal Society, the paper was referred, among others, to Sir Charles Wheatstone, who was my intimate personal friend. Wheatstone was an eminently fair man and a good judge, but the discovery did not then recommend itself to his mind. For a whole Sunday afternoon we walked on Barnes Common, discussing the experiments and their consequences, if true, to science. But all my arguments were insufficient to convince my friend; and I fear that then the Royal Society did not appreciate and publish the researches. I write from memory only, for I know that, later, no Society or institution honoured Joule more than the Royal Society and its members.

Not for one moment, however, did Joule hesitate in the accuracy of his experiments or his conclusions. He once suggested to me that we might take a trip together to the Falls of Niagara, not to look at its beauties, but to ascertain the difference of temperature of the water at the top and bottom of the fall. Of course the change of motion into heat was a necessary consequence of his views.

No more pleasant memory of my life remains than the fact that, side by side, at my lectures in the Royal Institution, used to sit the illustrious Dalton, with his beautiful face, so like that of Newton, and the keenly intelligent Joule. I can give no other explanation than the fact of organic chemistry being then a new science that two philosophers of such eminence should come to the lectures of a mere tyro in science. I used to look upon them as two types of the highest progress in science. Newton had introduced law, order, and number into the movements of masses of matter in the universe; Dalton introduced the same into the minute masses which we call atoms; and Joule, with a keen insight into the operations and correlation of forces, connected them together, and showed their mutual equivalence.

I do not know whether these memories are of any use to you, but, such as they are, they are at your disposal for your lecture on the friend of my youth. Yours sincerely,

LYON PLAYFAIR.

WEIGHING BY A SERIES OF WEIGHTS.

THIS subject, now under discussion in NATURE, is by no means new. The following remarks on the general theory of such questions may prove interesting. The problems are divisible into two classes:—

(1) To assign a series of weights so as to be able to weigh any weight of an integral number of pounds from 1 to n inclusive, the weights being placed in only one scale-pan.

(2) The same problem when the weights may be placed in both scale-pans.

Two other conditions may be imposed, viz.:—

(*a*) That no other weighings are to be possible.

(*b*) That each weighing is to be possible in only one way, *i.e.* to be unique.

The question considered in NATURE is of the second class, and is further subject to the two conditions above stated. Moreover, the problem for the number of pounds 1, 4, 13, 40, &c...., as solved by the series of weights 1; 1, 3; 1, 3, 9; 1, 3, 9, 27; &c...., respectively gives in each case the series containing the least number of weights for which the solution is possible.

For a number of pounds equal to 40, the solution may be given by means of a single algebraic formula, constituting an identity, viz.

$$(x^{-1} + 1 + x)(x^{-3} + 1 + x^3)(x^{-9} + 1 + x^9)(x^{-27} + 1 + x^{27})$$
$$= x^{-40} + x^{-39} + \dots\dots + x^{-1} + 1 + x + \dots\dots + x^{39} + x^{40}.$$

This is easily verified. When the left-hand side is multiplied out, the resulting powers of x are obtained by addition of certain indices, some positive and some negative. The indices in question are 1, 3, 9, 27, and, from the form of the factors, may in any single term be taken *either* positively or negatively. Thus one term on the left is $x^{-3} \,.\, x^9 \,.\, x^{27} = x^{-3+9+27}$, giving the term x^{33} on the right-hand side. The identity shows that every number from 1 to 40 inclusive can be thus composed by the numbers 1, 3, 9, 27, taken positively or negatively. It shows, moreover, since the coefficients of the several powers on the right are each unity, that the composition is unique. The identity includes also negative integers from -1 to -40. This is necessitated by the very nature of the question.

The above is one out of eight distinct solutions of the problem before enunciated. Before giving the other seven, it will be as well to show the principles of solution in general.

Consider, first, the problem of the first class in which weights can only be placed in one scale-pan. The number of pounds to be weighed being n, we must discover the ways in which it is possible to break up the expression

$$1 + x + x^2 + x^3 + \dots\dots + x^n$$

into similar expressions, for, as will be evident, each such factorization corresponds to a definite solution of the problem. By similar expression is to be understood one in which the coefficients are unity and the indices are in arithmetical progression.

At the outset may be remarked the trivial solution consisting of n ones corresponding to the unfactorized expression. There is one solution of this kind in respect of every number n.

Putting the expression in the form

$$\frac{1 - x^{n+1}}{1 - x},$$

it will be clear that if $n + 1$ be a prime number, the expression cannot be factorized in the required form. This is a consequence of the properties of prime roots of unity.

Hence, when $n + 1$ is prime, the only solution is obtained by taking n one-pound weights.

If, however, $n + 1$ be not prime, the number of solutions depends upon the composite character of $n + 1$. If $n + 1 = st$ where s and t are primes, we may write

$$\frac{1 - x^{n+1}}{1 - x} = \frac{1 - x^{st}}{1 - x} = \frac{1 - x^s}{1 - x} \cdot \frac{1 - x^{st}}{1 - x}$$
$$= (1 + x + x^2 + \dots + x^{s-1})(1 + x^s + x^{2s} + \dots + x^{st-s}),$$

a factorization of the required form.

Multiplying x^p in the first factor into x^{qs} in the second, we find that the number $p + qs$ is composed by p numbers equal to 1, and q numbers equal to s. In general, every number from 1 to n inclusive can be composed by means of $s - 1$ ones, and $t - 1$ s's. That is to say, that all numbers of pounds from 1 to n (where n is of the form $st - 1$) can be weighed, and in a unique manner, by means of $s - 1$ weights of 1 pound each, and $t - 1$ weights of s pounds each.

Another solution is obtained from the identity

$$\frac{1 - x^{n+1}}{1 - x} = \frac{1 - x^{st}}{1 - x} = \frac{1 - x^t}{1 - x} \cdot \frac{1 - x^{st}}{1 - x_t}$$

obtained from the former by interchange of s and t. These two solutions, together with the trivial one, constitute the complete set of three solutions. For brevity, these may be represented in the form—

$$1^{st-1},$$
$$1^{s-1}\,s^{t-1},$$
$$1^{t-1}\,t^{s-1},$$

where s^{t-1} means $t - 1$ s's, and so on.

From the above simple example it will be sufficiently clear that the number of solutions in the case of a given number n is entirely dependent upon the form of $n + 1$, when regarded as a product of prime factors. The number of solutions is not in all cases easy to determine, but some of the simpler results are interesting. Suppose the number $n + 1$ to be a product of k unrepeated prime numbers, and, under these circumstances, represent the number of solutions in regard to the number n by

$$[1^k].$$

Then it is easy to prove the relation—

$$[1^k] = 1 + k[1^1] + \frac{k(k-1)}{2!}[1^2] + \frac{k(k-1)(k-2)}{3!}[1^3] + \dots$$

to k terms,

and thence to calculate the values of $[1^1]$, $[1^2]$, $[1^3]$,... in succession. The series of numbers thus obtained commences 1, 3, 13, 75, 541,... where it will be noticed that the second number, 3, corresponds with the result obtained above for the case of $n + 1$, being of the form st, or the product of two unrepeated primes. It may be observed that this series of numbers is of great interest, and presents itself frequently in mathematics, notably in Prof. Cayley's "Theory of the Analytical Forms called Trees" (see vol. xiii. and subsequent volumes of the *Philosophical Magazine*, and "Collected Papers," No. 203).

A much simpler case to consider is that for which $n + 1$ is merely a power of a prime number. If $n + 1$ be the kth power of a prime, represent the number of solutions in regard to the number n by

$$[k].$$

A small amount of reflection shows the truth of the relation

$$[k] = 2[k-1];$$

whence

$$[k] = 2^{k-1}.$$

Turning now to the second class of problems, in which it is permissible to place weights in either or both scale-pans, it can be shown that the theory is not essentially different from that belonging to those of the second class.